Lecture 18: Torque and Statics

If we place a mass at the end of a rod and try to lift the mass from the other end, it is difficult. The longer the rod, the more difficult it is. In civil and mechanical engineering the analysis of rods, beams and trusses is essential to the design of structures, such as bridges, cranes etc. In rehabilitation and prosthetics it is essential to understand the forces that different parts of our body are subjected to. These forces depend not only of the magnitude of the weights the body supports, but also the points of application of the forces.

The key new quantity we need to understand is is called the torque, τ. A force, F, applied perpendicular to a horizontal rod of length, r, leads to a torque, $\tau = rF$. If the force is applied at an angle θ to the rod, then the magnitude of the torque is given by, $\tau = rFsin(\theta)$, which reduces to rF which $\theta = 90 degrees$. We also need to define the direction of the torque, ie. which way does is make a system rotate? To determine the direction of rotation, point the fingers of your right hand along the direction of r and then curl your fingers in the direction of F. The direction of rotation is defined to be the direction of your thumb! This is called the right hand rule. Counterclockwize rotation is positive, while clockwize rotation in negative.

Example A light flagpole of length $2m$ is attached to a house at an angle of 30degrees to the vertical. Find the torque experienced at the flagpole support due to a mass $m = 10kg$ hanging from the end of the flagpole.

Solution The angle between the force and the direction of the flagpole is 150 degrees, so the torque is given by,

$$\tau = 2 * 10 * 9.81 * Sin(30) = 98.1nm \quad (1)$$

The first important thing to note is that even though the sum of the forces is zero, an object may still rotate. In order to stop rotation, we must make sure that the sum of the torques is zero. We shall only consider motion in two dimenions, so we only need to consider one rotation axis, the axis which is perpendicular to the plane of rotation. In most cases we take the plane of rotation to be the x-y plane, so that the axis of rotation is along the z-axis. The formal statement of equilibrium for solid bodies such as rods and beams is,

$$\sum F_x = 0; \quad \sum F_y = 0; \quad \sum \tau_z = 0 \quad (2)$$